A verification of model of upwelling radiance above the sea surface*

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Upwelling radiance
Sea surface
Numerical model

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Abstract

A model of upwelling radiance above the sea, utilizing a combination of single and simplified multiple scattering effects, is considered as to its credibility. Some results of computations of vertical diffuse reflection of radiance are compared to the results obtained with Monte Carlo technique. The comparison shows quite a good agreement between both models.

In the series of papers [7–9] a mathematical model of upwelling radiance field above the sea has been developed. Basic part of this model (at least for marine optics) concerns the transformation of radiance entering water into radiance leaving the water vertically upward. This part is also the most difficult one for verification, because of almost total lack of suitable experimental data. It is caused, in turn, by necessity of very refined and sophisticated technics to measure the parameters needed in the model. Especially hard to measure are the parameters such as the ratio of backscattering to absorption and the shape of the scattering phase function in the water, as well as the directional distribution of the radiance falling on the sea surface.

In spite of the obstacles mentioned above, the author managed to conduct some testing of the credibility of solutions presented in the work [9]. It has been made by comparison of results of that work with results published in works by Gordon and the others [2, 4, 5], this last being obtained by simulation of the solution of Radiative Transfer Equation by a Monte Carlo technique.

The author's model [9] is assumed to be somewhat less accurate than the MC method and more exact than the two-stream model [6]. It consists of two parts

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combined together, of which the first one is the quasi-single scattering model [5] with part of forward scattered light assumed as not leaving the beam and with true backscattering phase function, while the second one is the multiple scattering model (using the invariance method [1]), with phase function of the water taken as sum of isotropic function and delta function in forward direction.

The two parts are assumed to contribute additively and entirely in the full effect of forming the upward radiance. Their contributions have been evaluated in dependence on the ratio of backscattering to absorption coefficients of the sea water.

Generally, the results obtained by the MC technique should be exact. Unfortunately, the only published relationship in the works mentioned [2, 4, 5], which uses the parameters in accordance with parameters of the model to verify, it is the relationship concerning to irradiances. So, exactness of the MC method is to some extent degraded by simplifying assumptions, introduced for transfer into the field of radiances.

The relationship, which connects the upward E_{uw} and downward E_{dw} irradiances underwater to the inherent optical properties of the water, is given as a reflection function just beneath the sea surface:

$$R(0)=E_{uw}(0)/E_{dw}(0) \tag{1}$$

depending on the ratio of backscattering coefficient b_b to absorptio n coefficient a

$$\eta=b_b/a\,. \tag{2}$$

The form of the reflection function has been found by means of the MC technique as [5]:

$$R=\sum_{n=0}^{N} r_n x^n\,;\; x=\frac{\eta}{1+\eta}\,, \tag{3}$$

where r_n are the coefficients computed separately for collimated irradiance incident on the sea surface from the zenith, and for completely diffuse incident irradiance.

In the first place we should find the relation of the reflection function (1) to the coefficient of vertical diffuse reflection of radiance, referred to as reflection coefficient W, defined in the work [9]:

$$W=L_{ua}/E_{da}\,, \tag{4}$$

where: L_{ua} – radiance leaving the water vertically, just above the sea surface; E_{da} – irradiance incident on the sea surface.

The simplification, allowing to calculate the upward radiance from the R function, is done by assuming that the upwelling radiance just beneath the sea surface is completely diffuse (*i.e.* by placing a hypothetical Lambertian reflector of albedo $R(0)$ just beneath the sea surface) [2, 4].

As one of the above functions (R) is defined underwater and the other (W) above the water, the transmission functions of water surface should be introduced twice. Let us denote the transmission for radiance passing the surface from the water to the atmosphere by T_{wa}^{L}, and the transmission for irradiance passing the surface from the atmosphere to the water by T_{aw}^{E}. The both transmissions will be expressed as follows:

$$T^L_{wa}=\frac{L_{ua}}{L_{uw}}=\frac{1-\rho_0}{n^2}\,;\; T^E_{aw}=\frac{E_{dw}}{E_{da}}=1-\bar{\rho}_\alpha\,, \tag{5}$$

where:

n – refraction index of the water,

ρ_0 – Fresnel reflection coefficient for incident angle $\alpha=0$,

$\bar{\rho}_\alpha$ – the same averaged over angular distribution of radiance falling on the sea surface.

The reflection coefficient W may now be written in the form:

$$W=L_{uw}\,T^L_{wa}\,\frac{T^E_{aw}}{E_{dw}}\,. \tag{6}$$

From the assumption of ideal diffusivity of upwelling radiance just beneath the sea surface it results:

$$L_{uw}=\frac{E_{uw}}{\pi}\,. \tag{7}$$

So, according to the definition (1):

$$W=\frac{1}{\pi}T^L_{wa}\,T^E_{aw}\,R\,, \tag{8}$$

and this is the relation which was needed for comparison of the results in the works [9] and [5].

More exactly, according to [2, 3], the right side of equation (8) should be multiplied by a factor $(1-rR)^{-1}$, taking into account the photons which interact twice with the hypothetical Lambertian surface ($r=N_2/N_1$, where N_2, N_1 – the number of photons which interact twice and once, respectively). Still, r is very small (approx. 0.06 [2.3]), so $rR \ll 1$ and the above effect can be neglected in not very exact calculations, as in here.

In the work [5] the coefficients r_m of equation (3) have been computed, separately for collimated and diffuse irradiance falling on the sea surface. In the case of collimated irradiance incident on the sea surface from the zenith, the expansion coefficients are as follows:

$$r_0=0.0001\,,\; r_1=0.3244,\; r_2=0.1425,\; r_3=0.1308\,. \tag{9}$$

They can also be used for the directions of incidence slightly different from the zenith, let us say up to about 20° [5].

In the case of incident distribution of completely diffuse irradiance the coefficients r_m are:

$$r_0=0.0003\,,\; r_1=0.3687\,,\; r_2=0.1802\,,\; r_3=0.0740\,. \tag{10}$$

According to the above data and using the equation (8), the reflection coefficients

W have been computed for collimated irradiance (W_S) and diffuse irradiance (W_D) respectively, and presented in the Figures 1 – 3. In all the cases only a smooth sea surface has been taken into consideration.

The continuous lines in all the figures fit to the data found from the model verified (henceforth referred to as the V-model, with coefficient W denoted as W^{V}). The dashed lines instead, fit to the values found on the base of the works [2, 4, 5], using the Monte Carlo technique (referred to, shortly, as the MC-model, with coefficient W denoted as W^{MC}).

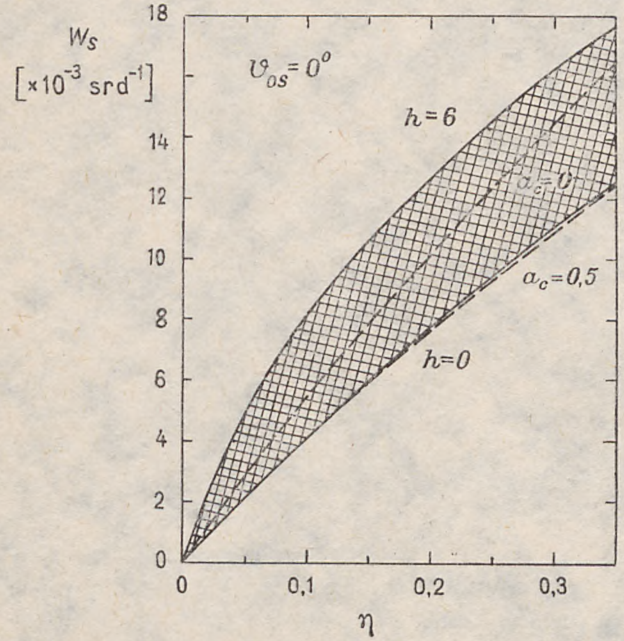

Fig. 1. Coefficient of vertical diffuse reflection of radiance W_S as a function of ratio of backscattering to absorption η, for collimated irradiance incident from the zenith
Continuous lines – functions $W_S^{\mathrm{V}}(\eta)$ computed from the model verified, for phase functions of water ranging from an isotropic ($h=0$) to a strongly stretched one in backward direction ($n=6$); dashed lines – functions $W_S^{\mathrm{MC}}(\eta)$ computed from the reference model for isotropic ($a_c=0$) and cardioidal ($a_c=0.5$) distribution of upward radiance underwater

Figure 1 presents a dependence of the reflection coefficient W_S on the ratio of backscattering to absorption η, found from both the models mentioned, for the case of collimated irradiance from the zenith. The dashed area comprises a set of possible values obtained from the V-model, in bounds determined for the scattering phase functions of water $p(\gamma)$, ranging from isotropic to stretched one in backward direction. Quantitatively the shapes of phase functions are defined by an anisotropy parameter h [9]:

$$h=\frac{p(180^\circ)-p(\gamma_{\min})}{p(\gamma_{\min})}, \tag{11}$$

where: $p(\gamma_{\min})$ – minimum value of $p(\gamma)$. In the case examined h ranges from 0 to 6.

The dashed line, which is comprised in the area mentioned, fits to the values obtained with the aid of the MC-model, assuming the ideal diffusivity of upward radiance underwater. The values of expansion coefficients (9) are found as average for several phase functions with $1 \leqslant h \leqslant 6$.

As mentioned, the drawing based on the MC-model does not transgress the area limited by lines of the V-model for $h=0$ and $h=6$, with tendency of passing from the $h=0$ line, for small η values, towards the $h=6$ line, for big η values. The tendency seems right and logic. Changes in the ratio of backscattering to absorption are possible theoretically with no changes of phase function's shape (*e.g.* with changes of concentration of suspended matter only). But still, and more probably in general, the growing of the ratio is possible which is caused, between others, by increase of the backward stretching of the phase function (changes of both concentration and size distribution spectrum of suspended matter). So, taking into account the simplifications mentioned, one can conclude quite a good, at least qualitative, agreement of both models in the examined case.

Quantitatively, the relative deviation from the mean of the two models ranges from about $\pm 35\%$ for smallest η, to $\pm 9\%$ for biggest η, which hardly may be regarded as limits of the measuring error.

The co-accordance improves radically if we get away from some a priori assumptions. Namely: 1) if h parameter is not literally regarded as an inherent optical property, but rather as the property averaged over some range of scatterance angle, and thus taking much lower values; 2) distribution of upward radiance underwater is not isotropic but rather cardioidal one:

$$L_{uw}(\vartheta) = L_{uw}\left(\frac{\pi}{2}\right)(1 - a_c \cos\vartheta)\,; \; 0 \leqslant a_c < 1\,, \tag{12}$$

which seems to be more realistic than isotropic case ($a_c=0$) [10]. In the range of $0 \leqslant h \leqslant 1$ it is possible to fit both models within limits of few percent of mutual deviation. For extremal instance, with $h=0$ and $a_c=0.5$ the lower limit of dashed area in the Fig. 1 means almost undistinguishly the same curve for both models.

The situation illustrated in Figure 2 looks similar to above. It presents courses of reflection coefficients W_D, found from the two models, for ideal diffuse radiance falling on the sea surface.

With initial assumption of ideal diffusivity of upward radiance underwater (upper dashed line, $a_c=0$) and $h=6$, disagreement of the courses compared does not exceed about $\pm 18\%$ of relative deviation from the mean. It can also be diminished to a few percent when cardioidal distribution of upward radiance and much lower values of h are introduced, as illustrated in the Figure by lower dashed and straight lines ($h=0$, $a_c=0.5$). In this, rather extremal case, the deviation does not exceed $\pm 3\%$, but it may be even better for some intermediate choosing of $0 \leqslant h \leqslant 1$ and $0 < a_c \leqslant 0.5$.

Another example of credibility of the V-model, with the MC-model as a reference, is presented in Figure 3. Here is shown a relation of the reflection coefficient

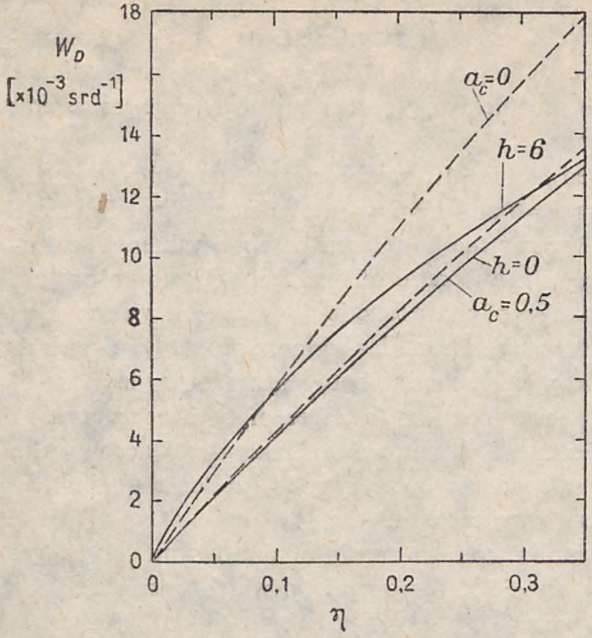

Fig. 2. Coefficient of vertical diffuse reflection of radiance W_D as a function of ratio of backscattering to absorption η, for completely diffuse incident irradiance
Continuous lines – functions $W_D^V(\eta)$ computed from the model verified ($h=0$ and $h=6$); dashed lines – functions $W_D^{MC}(\eta$ computed from the reference model ($a_c=0$ and $a_c=0.5$)

W_S on sun's zenithal distance ϑ_{os}, computed from both models. As, according to [5], the expansion coefficients (9) may be utilized for computation of R function in range of $0 \leqslant \vartheta_{os} \leqslant 20°$, the horizontal (dashed) lines are drawn in this range, fitting to the values of W_S^{MC} for several η values (isotropic case only, *i.e.* $a_c=0$). Likely to the case illustrated in Figure 1, the lines lay inside the dashed area, determined from the V-model for phase functions of water with anizotropy $0 \leqslant h \leqslant 6$. The $W_S^V(\vartheta_{os}, h)$ functions, computed for fixed values of h, also run almost horizontally, slightly increasing for $h=0$ (lower limit of the area mentioned) and decreasing not much for $h=6$ (upper limit).

The $W_S^{MC}(\vartheta_{os})$ functions show tendency similar to that in Figure 1. With increase of η they pass from lower to upper limit of area of possible W_S^V values, which, as previously, may be explained by great probability of simultaneous η and h increasing in natural conditions. The last effect is neglected in the MC-model, by averaging the influence of different phase functions on computations. It is good enough for finding the irradiance reflection function R, but may be giving a worse approximation for the radiance reflection coefficient W.

Summarizing, apart from the fact, that only two cases of incident radiance distributions were considered, the extremity of those (from collimated to isotropic one) allows to generalize the results of comparison, with some carefulness. So, one can

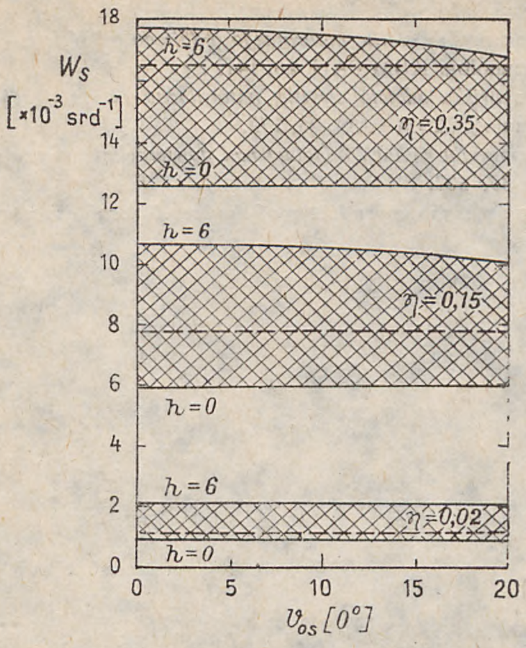

Fig. 3. Coefficient of vertical diffuse reflection of radiance W_S as a function of sun's zenithal distance ϑ_{os}, for several values of the ratio of backscattering to absorption η
Continuous lines – functions $W_S^{V}(\vartheta_{os})$ computed from the model verified, for phase functions of water ranging from an isotropic ($h=0$) to a strongly stretched one in backward direction ($h=6$); dashed lines – functions $W_S^{MC}(\vartheta_{os})$ computed from the reference model ($a_c=0$)

conclude a good qualitative agreement of the models discussed. Quantitatively, it may be even very good, when the inherent anizotropy parameter h of phase function of water is decreased by about one order of magnitude, for introducing it into the V-model, and when upward radiance distribution underwater in the MC-model has a minimum in nadir direction (the last being confirmed experimentally [10]). Of course, the conclusion does not exclude the need for further correction of the model verified, especially in pure experimental way.

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