Acoustic variability of central Bay of Bengal for tomography

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Acoustic tomography
Eigenfunction
Spatial variability
Evaporational cooling
Bay circulation

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Abstract

Empirical eigenfunction approach is used to determine the dominant sound velocity structure in the central Bay of Bengal. Eigenvectors have been derived from a set of two season hydrographic data to explore the horizontal and vertical variability of the acoustic field. The first three energetic eigenfunctions account for more than 99% of the variance in the data set. The first eigenfunction, which accounts for more than 97% of the variance, represents the mean sound velocity profile and its spatial variability. The second eigenfunction represents the layer of maximum variability and its spatial variations. The third eigenfunction represents the high-frequency fluctuation. This analysis brings out that the variability in the acoustic field is primarily controlled by the temperature field. However, in the upper layers salinity plays a dominent role. These statistical descriptions of sound speed variability are useful for mapping mesoscale variability in the Bay of Bengal using the techniques of acoustic tomography.

1. Introduction

Acoustic tomography, proposed by Munk and Wunsch (1979), is a tool for synoptic observation of ocean interior over large distances. This is achieved by measuring the travel times of acoustic signals between sources and receivers kept at suitable locations within ocean volume. These measured travel time data, along with theoretically computed travel times, can be made use of to reconstruct the temporally evolving sound speed field through geophysical inversion techniques. In order ot reconstruct the ocean variations from the observed travel time changes it is necessary to specify the models for both variations and their effect on travel times which, in turn, requires an *a priori* knowledge of the statistics of the sound speed field.

A knowledge of the hydrographic characteristics is a prerequisite for understanding of sound velocity structures. This is particularly true in the Bay of Bengal which is characterized by seasonal wind systems and immense land drainage. The seasonally reversing monsoonal circulation at the surface coupled with occasional cyclones during post monsoon period gives rise to highly complex and interdependent hydrographic parameters.

This paper is an attempt to investigate the high energetic modes of sound velocity field, in central Bay of Bengal, in relation to general hydrography of the region to provide base line information for an operational tomographic system.

Empirical eigenfunction (EEF) analysis has been widely adopted for statistical description of many types of distributions for the last two decades. It is important to point out that EEF's may or may not have physical significance/interpretation but depends entirely on the data examined. However, various researchers from different disciplines employed this technique and after careful analysis and interpretation assigned appropriate physical meaning to the different modes of EEF. For example, Stidd (1967) used eigenvectors to represent the seasonal variation of rainfall over Nevada and found that the first three eigenvectors in order of importance have features in common with the three natural cycles of precipitation. Winant *et al* (1975) and Aubrey (1978) used this technique to beach profile data and attributed the first three eigenvectors to mean beach profile, seasonal cycles, and high-frequency random oscillations, respectively. Ramana Murty *et al* used this method to grain-size data to identify the different energy environments along central west coast of India. More recently Rezzoli *et al* (1985) applied this technique for statistical description of sound speed variability in the Gulf Stream region to improve the resolution of stochastic inversion.

2. Material and methods

The desired hydrographic data from central Bay of Bengal have been collected during two cruises of r/v 'Gaveshani' (Fig. 1) representing onset (May 24—June 6) and receding phase of SW monsoon (September 12—October 8) of 1983. From observed temperature and salinity sound velocity has been computed following Chen and Millero (1977). These data have been further subjected to empirical eigenfunction analysis.

3. Computational procedure

The hydrographic data (temperature, salinity and sound velocity) were subjected to EEF analysis to examine the vertical and horizontal variability by setting the data matrix $\mathbf{H}_{m \times n}$ in such a way that each row (m) contains the data (physical property) at each of the stations at a particular depth and

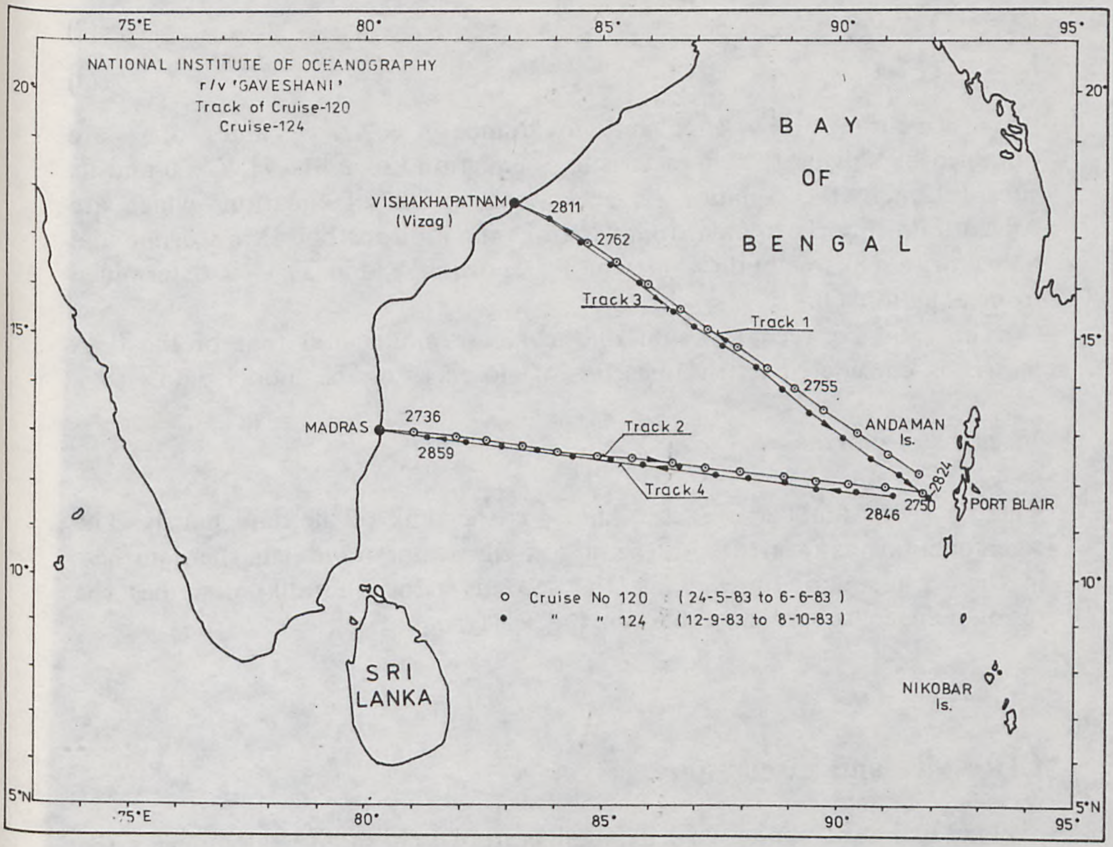

Fig. 1. Geographical locations of station positions

each column (n) contains the data (physical property) at each of the depth point at a particular station. The variability of the data matrix is explained in terms of a few eigenfunctions of the matrix **H**. This is achieved by normalising and decomposing **H** into a product of 3 matrices using SVD technique (Lawson and Hanson, 1974).

In the matrix form:

$$\mathbf{X}_{m\times n} = \mathbf{H}^{1/2}_{m\times n} = \mathbf{U}_{m\times r}\,\mathbf{\Gamma}_{r\times r}\,\mathbf{V}^{T}_{r\times n}, \tag{1}$$

where m is the number of standard depth points (column) and n is the number of stations at which data is taken (row), and T is the transpose of the matrix. The column vectors of **U** and **V** matrices are orthogonal, *ie*:

$$\mathbf{U}^T\mathbf{U} = \mathbf{I},\ \mathbf{V}^T\mathbf{V} = \mathbf{I}$$

and $\mathbf{\Gamma}$ is a diagonal matrix with diagonal elements called singular values of **H**, and $r\,[r \leqslant \min(m, n)]$ is the rank of the matrix **H**; **U**, $\mathbf{\Gamma}$, and **V** satisfying equation (1) are obtained by solving the eigenvalue problem:

$$(\mathbf{B}-\lambda\mathbf{I})\,\mathbf{V} = 0, \tag{2}$$

$$(\mathbf{A}-\lambda\mathbf{I})\,\mathbf{U} = 0, \tag{3}$$

where covariance $\mathbf{B} = \mathbf{X}^T\mathbf{X}$ and covariance $\mathbf{A} = \mathbf{X}\mathbf{X}^T$. Values of λ are obtained by solving the characteristic of equation (2), *ie* $|\mathbf{B}-\lambda\mathbf{I}|\mathbf{V} = 0$ and its substitution in the equation (2) leads to a system of equations which are solved following the standard Gaussian elimination method (Mc Cormic and Salvadori 1968) to obtain values of **V**. Knowing **V** and **Γ**, **U** is determined from equation (3).

The ratio between the sum of the factor model and that of the data matrix is considered as the measure of closeness of the model data:

$$\text{Measure of closeness} = \sum_{i=1}^{k} \lambda_i \Big/ \sum_{i=1}^{\gamma} \lambda_i,$$

where k is the number of factors and γ is the rank of the data matrix. The eigenfunctions associated with the largest eigenvalue represents the data best in the least square sense, while the second factor (in rank) describes the residual mean square data best in the least square sense.

4. Results and discussion

The EEF's of sound velocity, temperature and salinity along with the vertical cross-section of temperature and salinity are presented in Figures 2–9. The salient features of their distributions are discussed below. Since the contribution of the third eigenfunction is negligibly small (Table 1) they are not considered in this study.

Table 1. Percentage of mean square values of the sound speed data as explained by the first three eigen values

Phase	Eigenvalues		
	first	second	third
Vizag–Port Blair			
Onset phase	97.76	1.68	0.32
Receding phase	98.65	1.13	0.12
Madras–Port Blair			
Onset phase	98.57	1.15	0.10
Receding phase	98.78	0.56	0.33

4.1. Madras–Port Blair section

The first vertical eigenfunction (V1) of sound velocity which resembles the mean velocity profile – during the onset phase of monsoon (May 24 – June 6) shows that the mean velocity profile has an isovelocity layer up to a depth of 30 m (Fig. 2). From there onwards the velocity decreases rapidly till a depth of 200 m and beyond this depth decrease is gradual reaching a minimum between 1400 – 1800 m. Thereafter the velocity increases. During the withdrawal phase of monsoon (September 6 – October 8) also the mean profile is more or less similar with minor departure *viz* the isovelocity layer deepens to a depth of 40 m, the velocity values at the upper 50 m and below 200 m depth are slightly higher than those of the onset phase.

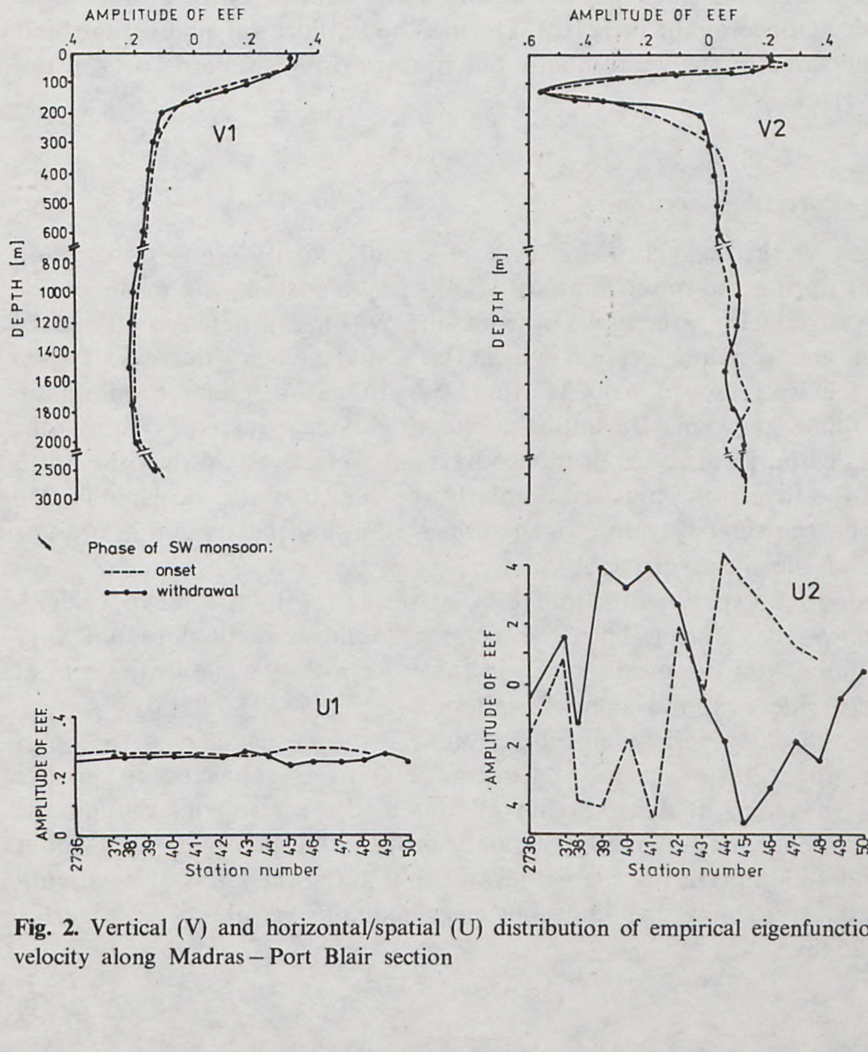

Fig. 2. Vertical (V) and horizontal/spatial (U) distribution of empirical eigenfunction of sound velocity along Madras – Port Blair section

The associated horizontal/spatial function (U1) shows that the sound velocity for both the periods in the western Bay of Bengal do not vary much from the mean profile. However, towards the eastern side the velocity values are higher than the mean profile value during the onset phase while during the withdrawal phase it is less.

The second vertical function (V2), in general, depicts the layer of maximum variability. Its distribution shows that during the onset as well as the withdrawal of south-west monsoon, the upper layer (up to a depth of 300 m) is subjected to greater variations in sound speed with maximum changes occurring at a depth of 125 m. In the deeper layers, however, the variability is much less though secondary peaks could be seen at a depth of 1000 m and 1800 m, respectively. Thus, this function indicates the layer of maximum variability and could be used in identyfying the tomographic layer.

The associated horizontal/spatial function (U2) shows that during the onset phase sound velocity (in the upper layers) slightly increases in the western side and decreases rapidly towards the central part. Towards the eastern side it once again increases. During the withdrawal phase the trend remains the same in the western side but in central and eastern parts of the Bay it reverses.

4.2. Vizag–Port Blair section

The first vertical eigenfunction (V1) of sound velocity along this section shows that during the onset phase of south-west monsoon, the mean profile has a thin isovelocity layer of 10 m from surface (Fig. 3). Between 10 and 20 m there is an inversion, beyond which the sound velocity decreases. Then onwards the decrease in velocity value is gradual reaching a minimum between 1400–1800 m. Beyond this depth velocity increases gradually. During the withdrawal phase of monsoon the isovelocity layer deepens up to 30 m and the inversion vanishes. Similarly the velocline also deepens to 250 m. However, the velocity values in the upper 50 m and below 200 m are less than that of the onset phase.

The horizontal/spatial function (U1), associated with the mean velocity profile, shows that during both the periods sound velocity does not vary much spatially from the mean profile in the eastern Bay, while in the central and western side it shows some variation.

The second characteristic eigenfunction (V2) shows that during the onset phase the upper layer up to a depth of 300 m is subjected to greater variability with maximum occurring at 100 m. Below 300 m, though the magnitude of changes are small, secondary peaks could be observed at about 1200 m and 1800 m. During the withdrawal phase the thickness of maximum variability layer remains the same but the maximum variability occurs at a depth of 75 m.

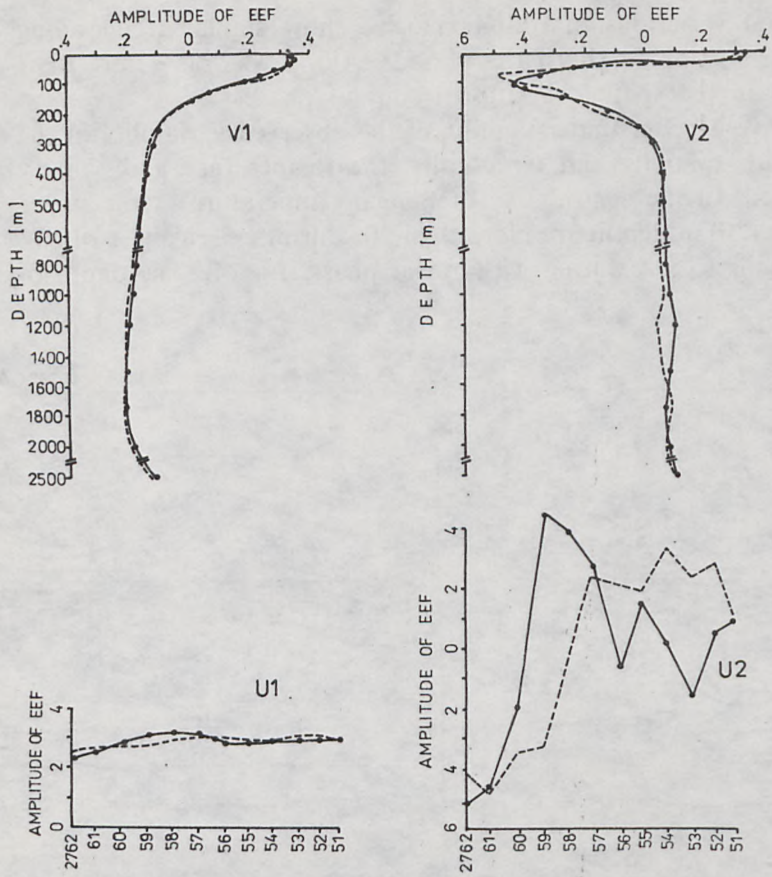

Fig. 3. Vertical (V) and horizontal/spatial (U) distribution of empirical eigenfunction of sound velocity along Vizag – Port Blair section (for explanations see Fig. 2)

The associated horizontal/spatial function (U2) shows that the sound velocity, in the upper layers, increases rapidly in the western Bay of Bengal during the onset phase while during the receding phase the increase is gradual and has greater spatial extend. Towards the east the trend shows oscillatory nature indicating slow decrease and increase in velocity for both the periods.

The salient features of the EEF analysis of sound velocity are as follows:

(i) Sound velocity values below 200 m along Madras–Port Blair section increases from onset phase to withdrawal phase. Along Vizag – Port Blair section sound velocity values decreases from onset to withdrawal phase.

(ii) The depth of maximum variability along- Madras – Port Blair section is 125 m for both the periods. Along Vizag – Port Blair section during the onset phase it is 125 m while during receding phase it shallows to 75 m.

(iii) The spatial variations of sound velocity show similar trend along Vizag – Port Blair section for both the periods. Along Madras – Port Blair section the trend in the spatial variation is out of phase.

In order to have a better understanding of the observed variability in the acoustic field, both spatially and temporally, the temperature and salinity fields are examined. In the central Bay of Bengal temperature in the upper layers (up to about 30 m depth) are more than 30° during onset phase of SW monsoon which falls to 28° during withdrawal phase (Fig. 4). The depth of

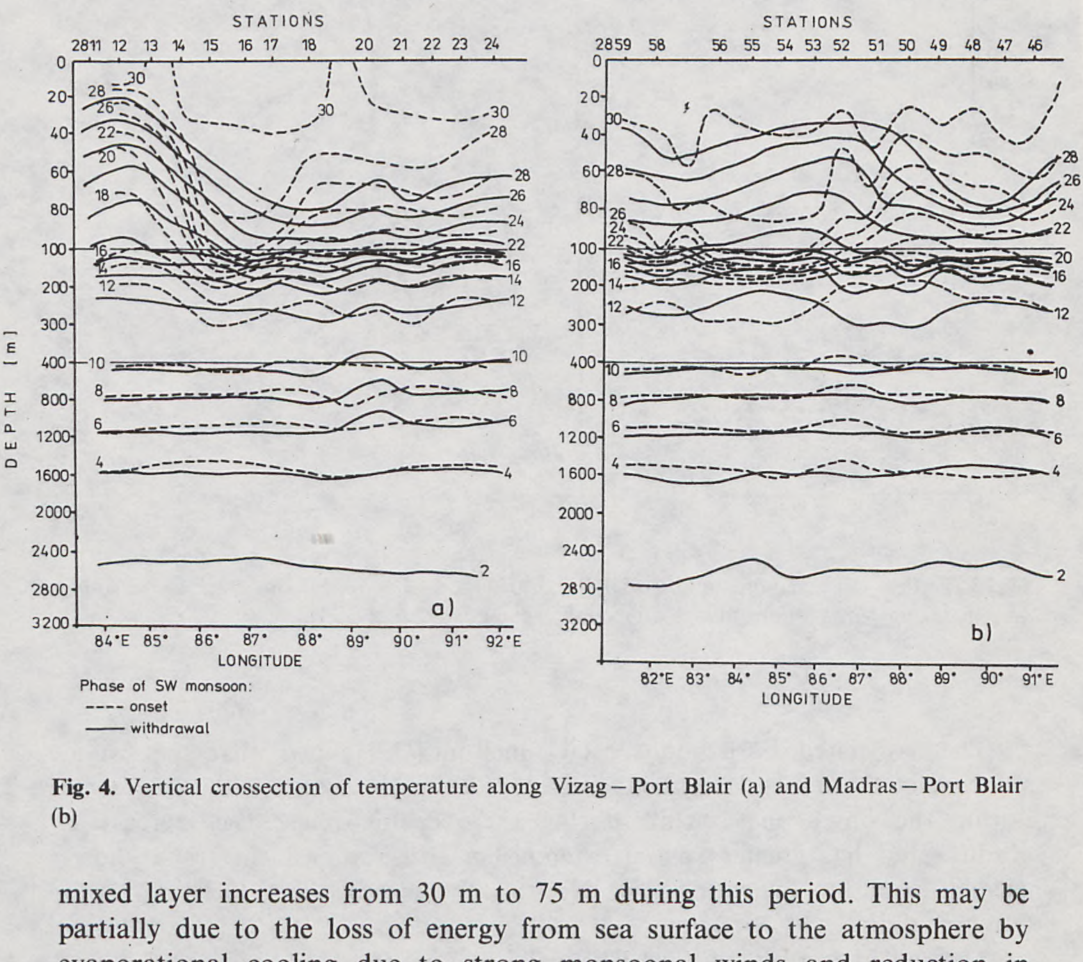

Fig. 4. Vertical crossection of temperature along Vizag – Port Blair (a) and Madras – Port Blair (b)

mixed layer increases from 30 m to 75 m during this period. This may be partially due to the loss of energy from sea surface to the atmosphere by evaporational cooling due to strong monsoonal winds and reduction in incoming solar radiation by clouds. A part of the heat may be transported into deeper layers due to vertical mixing and diffusion which is manifested by the deepening of mixed layer. The isotherms show an upward tilt off Vizag which is probably due to Ekman's drift forcing the cooler, denser subsurface water to come up under a northerly flow. This results in the lowering of sound speed values, as could be seen from Figure 3.

The first vertical function of temperature (V1) along Madras – Port Blair section shows an increase in temperature below 200 m from onset to withdrawal of SW monsoon (Fig. 5) while along Vizag – Port Blair the profile shows slight decrease in temperature during this period (Fig. 6). This, in turn, is responsible for the observed increase in sound speed along Madras – Port Blair and decrease along Vizag – Port Blair.

The vertical crossection of salinity in the central Bay of Bengal shows an increase in salinity in the upper 200 m and below this depth the salinity is more or less the same (Fig. 7). This is also evident from the distribution of V1 (Fig. 8, 9). Moreover, an examination of distribution of V2 of temperature and salinity shows that the top 200 m layer is subjected to the maximum variability. This reflects that Bay of Bengal can be visualised as a two-layer system from salinity-temperature distribution.

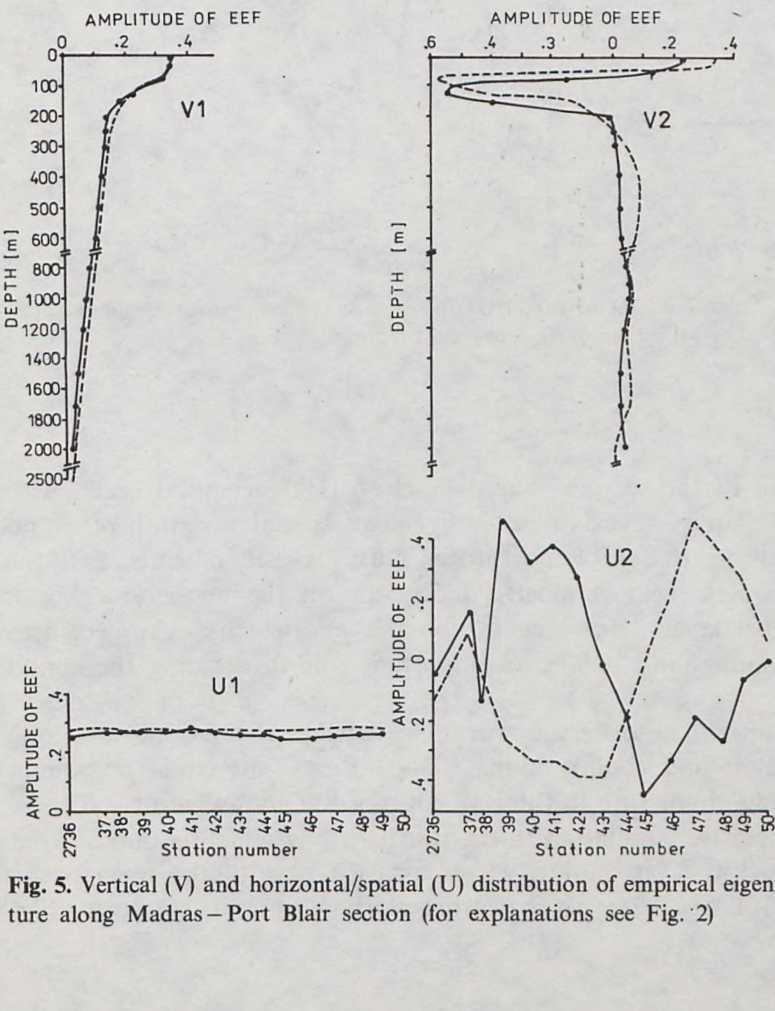

Fig. 5. Vertical (V) and horizontal/spatial (U) distribution of empirical eigenfunction of temperature along Madras – Port Blair section (for explanations see Fig. 2)

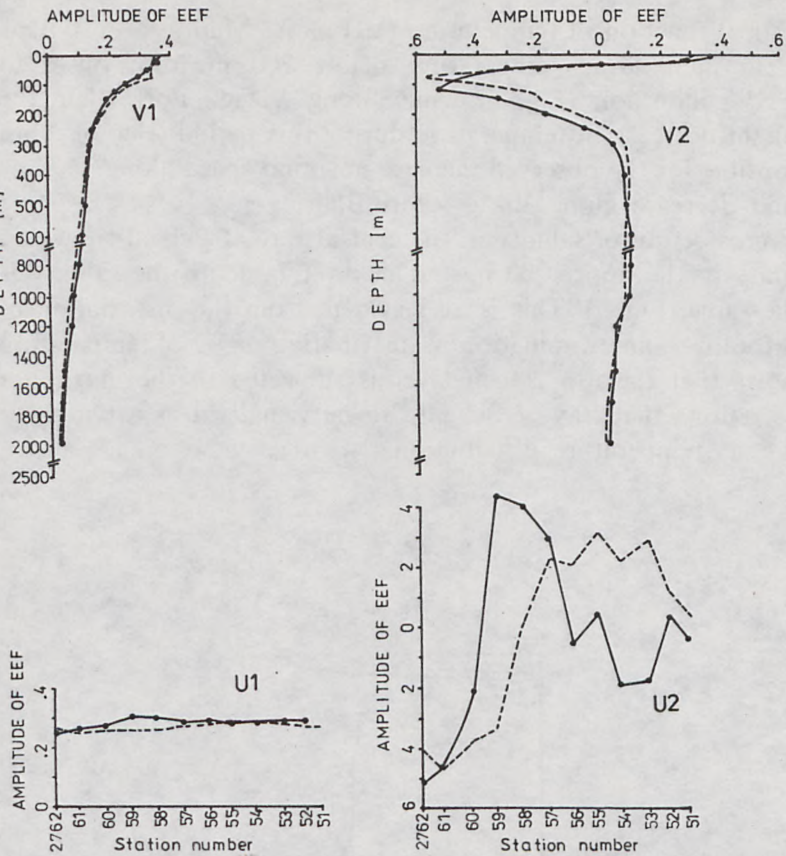

Fig. 6. Vertical (V) and horizontal/spatial (U) distribution of empirical eigenfunction of temperature along Vizag – Port Blair section (for explanations see Fig. 2)

A comparison of the second spatial function (U2) of sound speed with temperature and salinity reveals that the trend of spatial variation of sound velocity is similar to that of temperature. This suggests/indicates that the variability of acoustic field is mostly dependent on the temperature field, especially in deeper layers. However, in upper layers (which are subjected to maximum variation) salinity plays an important role in deciding the sound speed.

An explanation to the observed warming of water (below 200 m) along Madras – Port Blair and cooling along Vizag – Port Blair could be sought from the prevailing circulation in the Bay. During SW monsoon period there exists an anti-clockwise circulation in the northern bay which is purely wind driven and a clockwise circulation which forms a part of the open ocean circulation (Fig. 10). According to the earlier work (Varkey, 1986) during SW

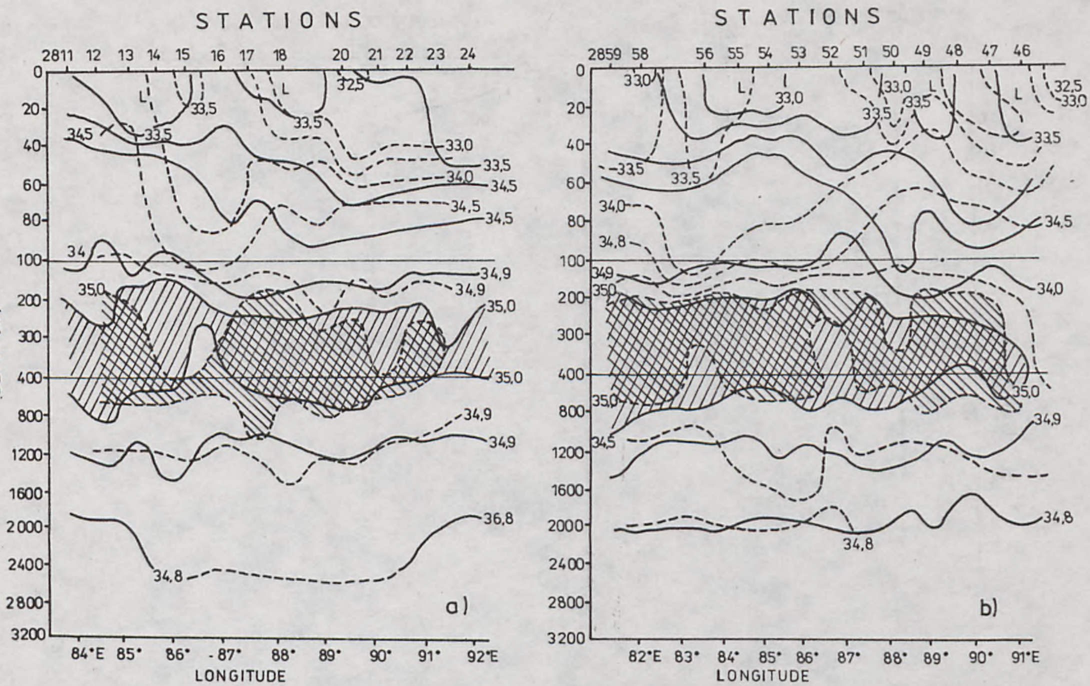

Fig. 7. Vertical crossection of salinity along Vizag – Port Blair (a) and Madras – Port Blair (b) (for explanations see Fig. 4)

monsoon period there is a net inflow of water into the Bay (considering 6° N Lat. as the outer boundary). Water entering into the Bay through the surface layers sinks to deeper layers under the influence of the clockwise gyre as could be seen from the warming of the deeper layers along Madras – Port Blair section. However, the observed cooling along Vizag – Port Blair indicates that the above mentioned phenomena may not be extending to the northern bay where a reverse process seems to be operative under the influence of the wind driven gyre.

5. Conclusion

This study suggests that the first two high energetic static modes (eigenfunction) are sufficient to understand the distribution of acoustic field and its spatio-temporal variations, in the central Bay of Bengal, for the future tomographic experiments in this region. The first eigenfunction very well represents the mean acoustic field which can be considered as the background profile while the second one clearly demarcates the layer of variability.

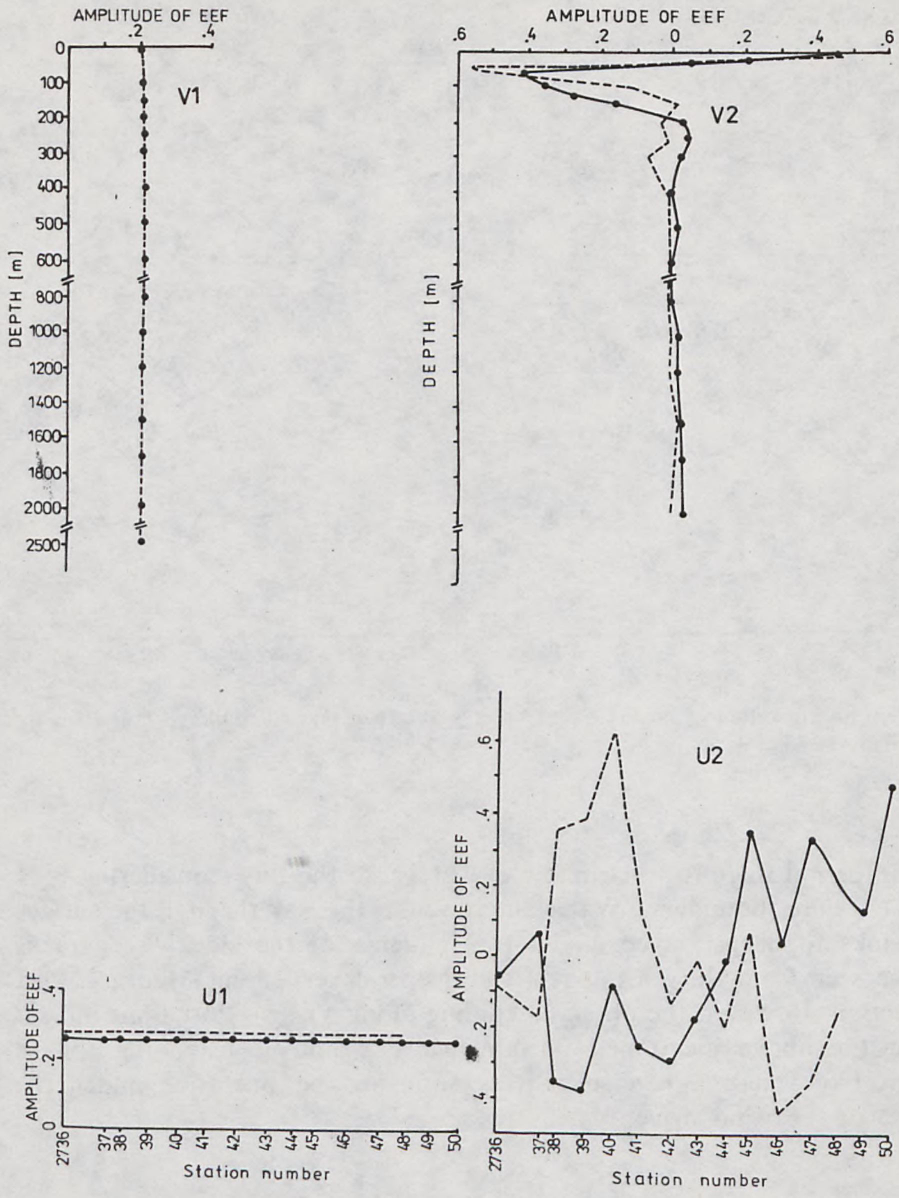

Fig. 8. Vertical (V) and horizontal/spatial (U) distribution of empirical eigenfunction of salinity along Madras – Port Blair section (for explanations see Fig. 2)

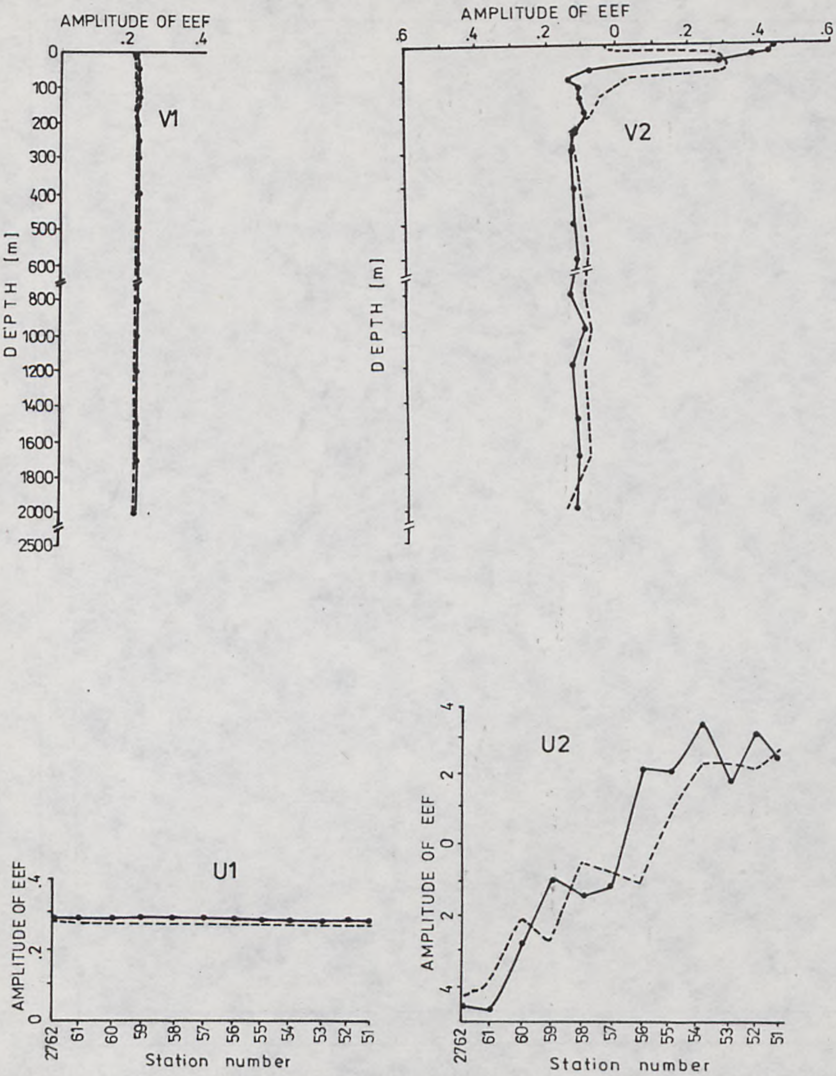

Fig. 9. Vertical (V) and horizontal/spatial (U) distribution of empirical eigenfunction of salinity along Vizag – Port Blair section (for explanations see Fig. 2)

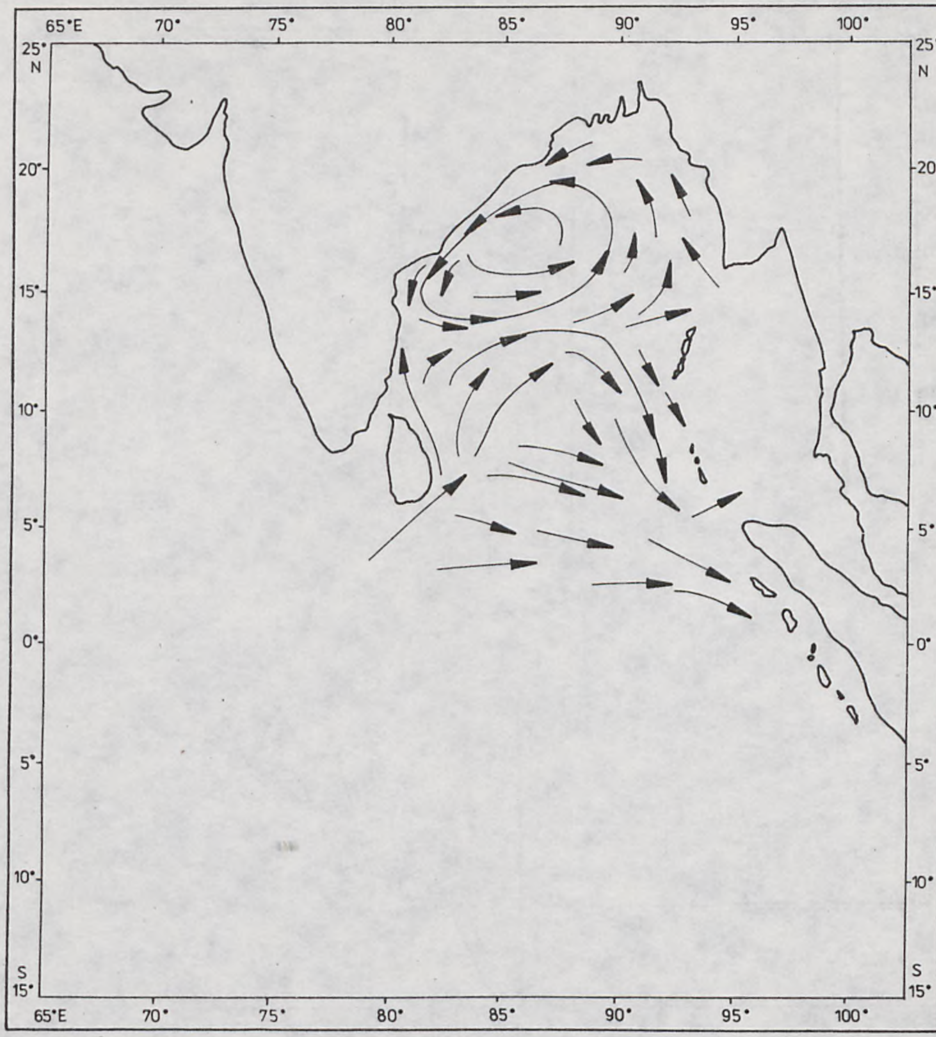

Fig. 10. Schematic diagram of surface currents in south-west monsoon (after Varkey, 1986)

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